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Pairing interaction and reaction mechanism for one- and two-particle transfer reactions: a simple model in one dimension

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Abstract. We discuss the reaction mechanism associated with two-particle transfer reactions in a simple one-dimensional model. The reaction process is generated by two colliding wells and we follow in time the evolution of the two-particle wave function, initially concentrated in one of the two wells. At the end of the process one can single out the population of the different final channels, including one and two-particle transfer. When a residual short-ranged pairing interaction among the two particles is included in addition to the moving potentials, one observes a clear enhancement of the pair transfer as compared to the expectation of a pure sequential one-particle transfers. The final "exact" solution can be compared to the one obtained within different reaction and structure models (as coupled-channels, first-order approximation, approximate treatment of the continuum, etc), so providing important information on the reaction mechanism associated with the different processes.

Introduction

The main goal of this contribution is the description of a simple model to describe both structure and dynamics of weakly-bound systems with one or more valence particles. Even considering inert cores, the problem is relatively easy only with one valence particle (one-particle halo), but starts to be more complex with two particles (two-particle halo), becoming extremely complicated for systems with more active particles. For these reasons one typically uses the expedient of resorting to a number of reaction models and approximation schemes (coupled-channels, first-order approximation, space truncation, effective optical potentials and formfactors, etc) that need to be tested (not only against experimental data). Particularly relevant in the case of weakly-bound systems is the treatment of continuum states and the associated procedures of continuum discretization. In addition, one would like to shed some light on the so-called reaction mechanism, namely on the description of the process in terms of single or repeated action of the external field in a perturbative expansion. A typical example is provided by the two-particle transfer process. Is the pair transferred in a single shot or in a correlated sequence of two single-particle transfer through a number of intermediate states? To make feasible the solution of the problem, we will simplify it by assuming particles to move just in a one dimension. In spite of the drastic assumption, the problem may maintain the main features and properties of the full three-dimensional case, for example for the description of transfer, inelastic and break-up processes. In particular this choice will allow us to treat in a simple way the action of the pairing correlations and to clarify their connection with two-particle transfer or two-particle break-up processes. Preliminary applications of the model to one and two-particle systems can be found in Refs.[1]

Reactions involving one particle

We first consider processes involving just one active particle, initially sitting on a single-particle level of a one-body potential and feeling the action of a second moving potential, as esemplified in Fig. 1.

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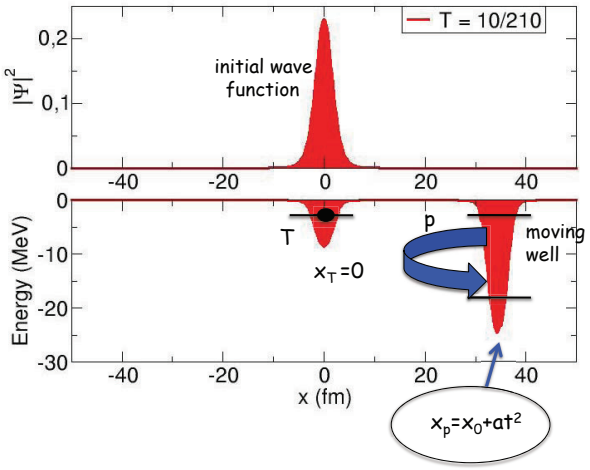

FIGURE 1. Schematic picture of the process. The upper frame gives the initial wave function probability, while the fixed and moving wells are shown in the lower frame.

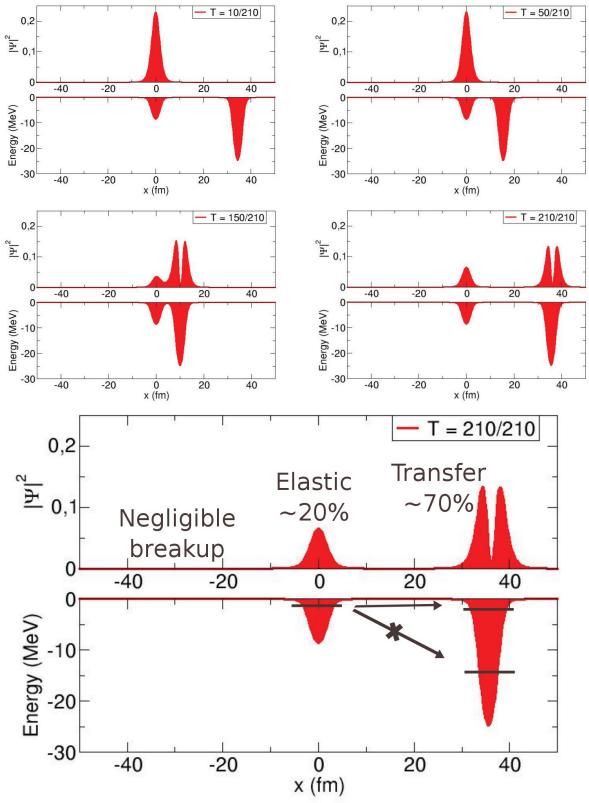

FIGURE 2. Evolution of the single-particle wave function (and corresponding evolution of the two wells) at different times. The final situation is better shown in the enlarged frame at the bottom of the figure. The parameters of the initial well have been chosen to create an intial well-bound state (E_b=-3.10 MeV).

We have therefore to solve the single-particle time-dependent equation

$$i\hbar\frac{d}{dt}\Psi(x,t) = \mathcal{H}(x,t)\Psi(x,t) \quad \text{with } \mathcal{H}(x,t) = -\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + V_T(x) + V_P(x,t). \tag{1}$$

The choice of the parameters entering in the calculation will lead to different structural and kinematical conditions, corresponding to rather different physical situations and simulating different bombarding energy regimes, different impact parameters, different Q-values for particle transfer: essentially one has to fix the parameters characterizing the two wells (consequent energies of single-particle states in both potentials), initial condition (selecting one of the single particle state in target potential), distance of closest approach x_0 and finally acceleration at x_0.

As an example of the evolution of the wave function we chose the fixed well in such a way that the single particle is sitting in the only well-bound state (with binding energy E_b=-3.10 MeV), while the moving well admits two (initially empty) bound levels, with the second level having the same binding energy as the state in the first well. For the parameters defining the trajectory we have taken x_0= 10 fm and a= $\frac{1}{2}$0.6fm·$\hbar^2$/ps^2. The evolution of the wave function with time is illustrated in Fig. 2. The different frames refer to different times (the total collision time is divided in 210 steps and the corresponding time is quoted in each frame), and in each frame the upper part gives the square of

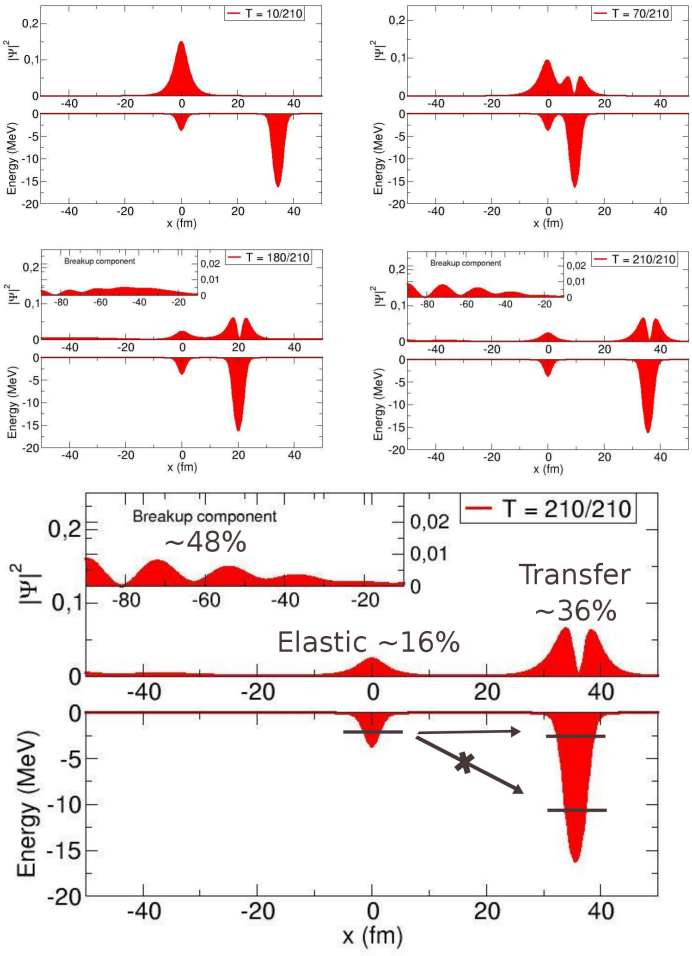

FIGURE 3. Evolution of the single-particle wave function (and corresponding evolution of the two wells) at different times. The final situation is better shown in the enlarged frame at the bottom of the figure. The parameters of the initial well have been chosen to create an intial weakly-bound state (E_b=-0.91 MeV).

the one-particle wave function while the lower frame gives the actual position of the two potentials at the same time. As apparent from the figure, when the tail of the moving well starts to overlap with the fixed well (third frame) part of the wave function enters in the moving well and then follows its movement. At the end of the process, by taking the overlap of the final wave function with the initial wave function and the wave functions of the single-particle states of each well one can determine the probability of elastic scattering and of one-particle transfer process. The presence of a node in the part of wave function inside the moving well already clearly indicates that the transfer takes place to the second single particle state (corresponding to a final channel with Q≈0). Note that only a negligible part of the wave function (practically undetectable in the figure) appears outside the two wells, indicating a rather small break-up probability (≈ 10%).

As a second example we chose an initial well producing a weakly-bound state (E_b=-0.91 MeV). The corresponding evolution of the wave function is shown in the different frames of Fig. 3. The weak-binding situation leads to an initial wave function with a longer tail than in the previous case. As a consequence, part of the wave function is already trasferred to the second well even before the overlap of the two wells (second frame). At the end of the process there is a large transfer probability, but the weak-binding situation has also led to a large fraction of the wave function outside of the two wells, therefore associated to large break-up processes.

The relative importance of the different final channels (elastic, inelastic, transfer, break-up) can be altered by changing the different parameters entering in the model. We only mention here that the transfer probability is strongly dependent on the single-particle energies generated by the two wells, which give rise to the Q-values for the different transfer channels. It can be shown that our calculations as a function of the Q-value [1] reproduces the well-known rule that the transfer probabilty of a neutral particle (as in our case) display a gaussian behavior around the optimal Q-value $Q_{opt} \approx 0$.

The results shown so far have been obtained by directly solving the time-dependent one-particle Schroedinger equation. The same equation can be solved within the standard time-dependent coupled-channels formalism by constructing the non-diagonal transfer formfactors and expanding the wave function into the dual basis associated with the two wells (cf. Ref. [2]). One can in this way test the validity of the first-order approximation and the necessary truncation in the basis. This latter point is particular relevant in the case of weakly-bound systems in connection with the treatment of the positive-energy part of the spectrum and the procedures used for the discretization of the continuum. All these aspect will be discussed in a forthcoming paper [3].

Two-particle transfer processes and pairing interaction

Next step will involve the extension to the case of two-particle sytems and two-particle transfer processes. In this case we will have an initial two-particle state generated by the fixed well and one will follow the time evolution of the two-particle wave function due to action of the moving one-body potential. In addition there is the action of a residual pairing-like interaction between the two particles. We have therefore to solve the equation

$$i\hbar\frac{d}{dt}\Psi(x_1,x_2,t) = \mathcal{H}(x_1,x_2,t)\Psi(x_1,x_2,t) \tag{2}$$

with

$$\mathcal{H}(x_1,x_2,t) = -\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dx_1^2}+\frac{d^2}{dx_2^2}\right) + V_T(x_1) + V_P(x_1,t) + V_T(x_2) + V_P(x_2,t) + V_{\rm int}(x_1,x_2). \tag{3}$$

The residual pairing interaction is assumed as a density-dependent delta interaction $V_{\rm int}(x_1,x_2) = -V[\rho[(x_1+x_2)/2\rho_0]\delta(x_1-x_2)$, acting therefore only when the two particles are both inside the same well. We first consider the case in which the pairing interaction is switched off (uncorrelated case). In this case both particles will initially be sitting in an uncorrelated way in one single-particle state. An example of the corresponding wave function is given in Fig. 4 (upper-left frame) as a contour plot as a function of x_1 and x_2. Note that in this case the wave function shows equal probabilities for the two particles to be on the same side of the potential (cluster-like configuration) or on opposite sides (cigar-like configuration). We then follow in time the two-particle wave functon according to the time-dependent Schroedinger equation. The upper-right frame displays the wave function at the end of the process. From this wave function we can separate different final states: elastic/inelastic (both particles still in the initial well), one-particle transfer (one particle in the initial well and one in the moving one), one-particle break-up (one particle in the continuum outside the wells and one in the initial or final well), two-particle transfer (both particles in the moving well) and finally two-particle break-up (both particles outside the wells). In this specific case break-up processes (both

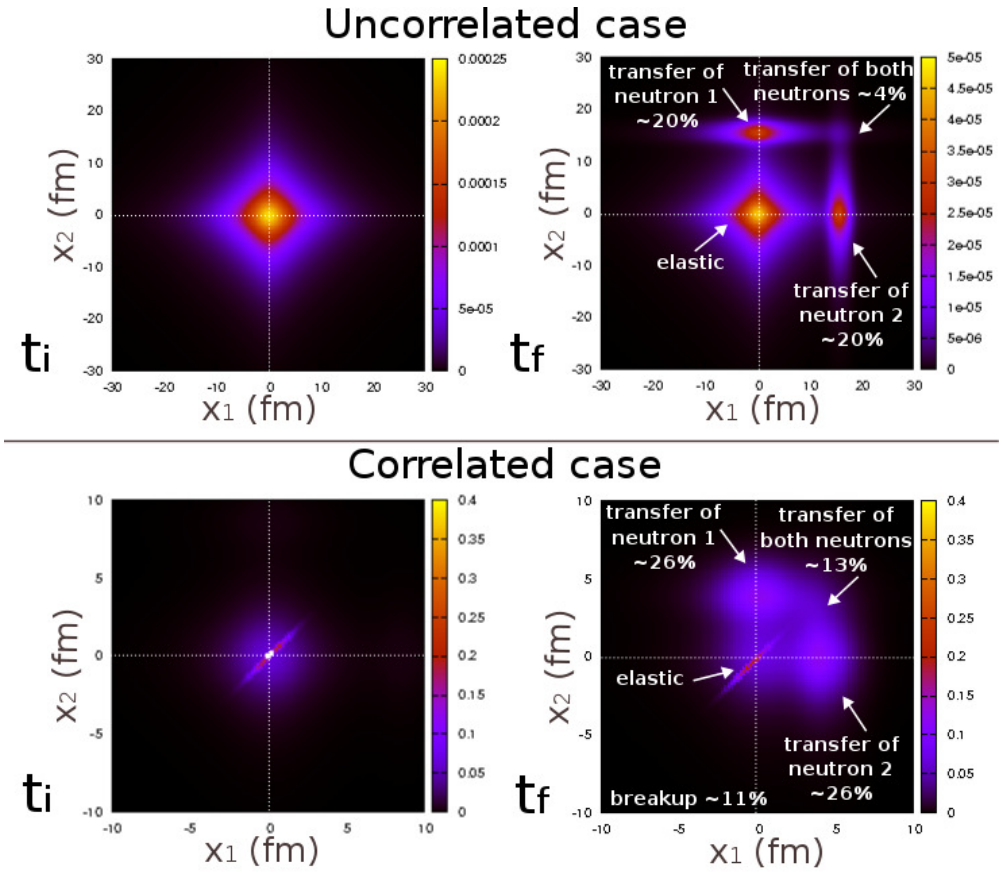

FIGURE 4. Square of the two-particle wave function shown as a contour plot as a function of x_1 and x_2. The four frames refer to the uncorrelated (upper part) and correlated case (lower part). Initial wave functions in the left column, final ones in the right column.

one and two-particle) are negligible. The total one-particle probability P_1 amounts to about 40%, while the two-particle transfer probability P_2 amounts to about 4%. Due to the absence of correlations the transfer process is induced by the one-body mean-field generated by the moving well, and in terms of reaction mechanism the two-particle transfer can only be interpreted as produced by the successive transfer of single particles. In such a situation, in a perturbative approach, we expect a pair transfer probability $P_2 \approx (P_1)^2/4$, which is precisely the value obtained in our calculation.

We switch now to the case with correlations. The corresponding initial and final wave functions are shown in the lower frames of Fig. 4. The initial wave function has been obtained by diagonalizing the residual pairing interaction in the two-particle basis. Continuum states have been included by a discretization procedure (cf. Ref.[4]). Note that due to the correlation the probability of finding both particles on the same side is now clearly favored. The effect of this initial correlation will propagate during the scattering process and affect the final wave function (lower-right frame). As in the previous uncorrelated case we can separate the probabilities for the population of the different final channels. One gets a total single-particle probability P_1 equal to 52 % and a pair transfer probability P_2 equal to 13%. This latter value is a factor 2 larger than the uncorrelated estimate $P_2 \approx (P_1)^2/4$. This factor 2 rapresents therefore the enhancement factor due to the pairing correlation. In order to clarify the reaction mechanism, next step will be to describe the processes within a coupled-channels scheme and verify that the same final result can be obtained within a model that includes the coherent contribution of successive one-particle transfers via the full set of levels in the intermediate one-particle system (continuum states included).

REFERENCES

[1] Dasso C H and Vitturi A 2007 *Nucl. Phys.* A **787** 476; Dasso C H and Vitturi A 2009 *Phys. Rev.* C **79** 064620; Moschini L 2013 *arXiv*1410.7167; Moschini L 2014 *J. Phys.: Conf. Ser.* **566** 012027; Vitturi A and Moschini L 2015 *J. Phys.: Conf. Ser.* **590** 012007 Hagino K, Vitturi A, Pérez-Bernal F and Sagawa H 2011 *J. Phys. G: Nucl. Part. Phys.* **38** 015105

[2] Esbensen H, Broglia R A and Winther A 1983 *Annals of Physics* **146** 149

[3] Moschini L etal., to be published

[4] Pérez Bernal F and Vitturi A 2009 *AIP Conf. Proc.* **1165** 305; Vitturi A and Perez Bérnal F 2010 *Nucl. Phys.* A **834** 428c; Vitturi A and Perez Bérnal F 2012 *AIP Conf. Proc.* **1488** 220